FPP Midterm Report

Yale-NUS College AY 17/18

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# Table of Contents

[1. Table of Contents 2](#_Toc495589360)

[2. Introduction 3](#_Toc495589361)

[3. Left-balanced binary trees 3](#_Toc495589362)

[3.1 Proposition: projecting\_an\_embedded\_binary\_tree 3](#_Toc495589363)

[3.2 Proposition: projectable 3](#_Toc495589364)

[4. Product of three consecutive numbers 4](#_Toc495589365)

[5. Mystery functions 5](#_Toc495589366)

[5.1 there\_is\_at\_most\_one\_mystery\_function\_k 5](#_Toc495589367)

[5.2 there\_is\_at\_least\_one\_mystery\_function\_k 5](#_Toc495589368)

[5.3 there\_is\_at\_most\_one\_mystery\_function\_l 5](#_Toc495589369)

[5.4 there\_is\_at\_least\_one\_mystery\_function\_l 5](#_Toc495589370)

[5.5 there\_is\_at\_most\_one\_mystery\_function\_m 5](#_Toc495589371)

[5.6 there\_is\_at\_least\_one\_mystery\_function\_m 5](#_Toc495589372)

[6. Conclusion 6](#_Toc495589373)

[7. Acknowledgements 6](#_Toc495589374)

# Introduction

In this report I explain the approaches taken and learning points gained from the process of solving each assignment part. Assignment specifications can be found at:

<http://users-cs.au.dk/danvy/YSC3236.2017-2018_Sem1/lecture-notes/week-07_midterm-project.html>.

# Left-balanced binary trees

## 3.1 Proposition: projecting\_an\_embedded\_binary\_tree

I was initially stuck as the specific unfold lemma unfold\_project\_Node\_Tree\_Leaf I was trying to write was too strong: it assumed that project\_binary\_tree\_into\_left\_binary\_tree t1 returned Some lt1 instead of None which is unprovable without any additional propositions. I tried to use the fact that t1 is left balanced, but then realized that this was identical to the second proposition projectable. The solution was simply to write unfold\_project\_Node\_Tree\_Leaf with the entire match expression; the induction hypothesis provided the fact that projecting an embedded lt returns Some lt, and this could be directly applied to the match expression in the goal.

## 3.2 Proposition: projectable

**Explanation for my original, indirect proof:**

The main challenge was trying to write a lemma about project\_binary\_tree\_into\_left\_binary\_tree (Node t1 n Leaf). As it turned out the lemma project\_t1\_with\_existential\_lt1 was kind of like a paraphrase of the original function, and was easy to prove after remembering the tactic destruct which helped me get rid of the existential in the assumption (this helped in other proofs I had been stuck in, such as in Part 2).

The tricky part was realizing that I had to define two distinct existentials in the lemma (and separate their scope via parentheses, otherwise the first existential applies to the second proposition as well). Previous iterations (see project\_t1\_with\_existential\_lt1\_initial) involved the single existential lt1 shared across both propositions, which seemed to make sense because in the definition, we have the same lt1 in both the match and the result. However as I realized eventually I was not able to apply the lemma in the inductive hypothesis (Error: Statement without assumptions) because the inductive hypothesis contained a single existential over its entire proposition, whereas the single existential in the lemma was applied over both propositions, which made it a single statement with no separate assumptions. The separate existentials in my final lemma ensured the inductive hypothesis matched the first proposition in the lemma exactly.

**Explanation for canonical, more direct proof (after Restart):**

The canonical proof involves no other lemmas besides the basic unfold lemmas. Instead of proving that t2 must be a Leaf, we consider the cases where t2 is a Leaf or Node; when it is a Node we know that Node t1 n t2 cannot be left balanced, and we use discriminate on the false proposition. In the case that it is a Leaf, instead of writing a lemma about project\_binary\_tree\_into\_left\_binary\_tree (Node t1 n Leaf) that the inductive hypothesis may be applied to, we rewrite the goal into a match expression involving just t1 using unfold\_project\_Node\_Tree\_Leaf, and then directly apply the inductive hypothesis on t1 (given the assumption that t1 is left-balanced, and after separating the existential from H\_t1 via destruct) on the match expression in the goal. This is actually similar to the approach taken in Proposition: projecting\_an\_embedded\_binary\_tree. The reason why I did not complete this simpler solution initially was because, although I had attained the assumption I wanted to apply directly to the match expression, I had not remembered about destruct at that point, and thus could not apply the assumption to the match expression as it contained an existential.

# Product of three consecutive numbers

I proved this mathematically rather than computationally. Writing a proof out on paper helped. The destruct tactic was crucial in allowing me to apply the inductive hypothesis. The lemma SSSn\_is\_3\_plus\_n made the main proof more elegant as it targets only the argument that I need to rewrite; otherwise I would have rewritten the other two arguments as well into arabic form, which would mean having to rewrite them back into Coq's successor form before applying the inductive hypothesis.

This proof is similar to the proof of the proposition that the product of two consecutive numbers is even, involving the same approach in the inductive case: Expand using the last term (n + k) to get an addition of two parts, the first part matching the terms in the inductive hypothesis and the second part being a multiple of k. Then factorize both by k.

# Mystery functions

## 5.1 there\_is\_at\_most\_one\_mystery\_function\_k

Since the Fibonacci secquence is a recurrence relation involving the previous two terms (rather than just the previous term), we would need to define an induction principle which uses two inductive hypotheses instead of just one. In this case, since the recurrence relation in specification\_of\_the\_mystery\_function\_k contains mf (S (p + q)), which can be applied to f (S (S n)), we know that the inductive step should be to P (S (S n)). In addition the LHS of the recurrence relation contains mf (S p) and mf (p), hence the inductive hypotheses should be P (n) and P (S n).

## 5.2 there\_is\_at\_least\_one\_mystery\_function\_k

Arithmetic manipulation.

## 5.3 there\_is\_at\_most\_one\_mystery\_function\_l

We need to define a different inductive hypothesis in this case; since the recursive relation in specification\_of\_the\_mystery\_function\_l contains mf (S (S (S n'''))), we know that the inductive step should be to P (S (S (S n))). In addition the other two terms of the recurrence relation are mf (n’’’) and mf (S (S n’’’)), hence the inductive hypotheses should be P (n) and P (S (S n)).

## 5.4 there\_is\_at\_least\_one\_mystery\_function\_l

Have not attempted.

## 5.5 there\_is\_at\_most\_one\_mystery\_function\_m

Proving this via induction is impossible as the specification is not inductive, and we cannot use an inductive hypoyhesis. We could, however, define another specification that *is* inductive, prove that there is at most one mystery function that satisfies that specification, and then prove that it is equivalent to specification\_of\_the\_mystery\_function\_m.

## 5.6 there\_is\_at\_least\_one\_mystery\_function\_m

The mystery\_function\_m and mystery\_function\_m\_aux flattens a tree to the right via the use of an accumulator. The accumulator enables the recursive function to achieve a transformation that involves successive ‘re-shuffling’ or re-constructing of the object to be returned (instead of inductively constructing at return time). The proof was solved via basic unfold lemmas.

# Conclusions

* Sometimes the solution is quite simple. So always make sure to write the basic unfold lemmas and do the canonical approach first.
* Pen and paper, as always.
* Name everything.
* To achieve optimal progress I need to spend a bit of time every day on FPP, attempt other assignment questions if I get stuck, and send questions at regular intervals.

# Acknowledgements

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